previously, we have introduced complex power:

$$S = P + jQ$$

$|S|$: apparent power — VA

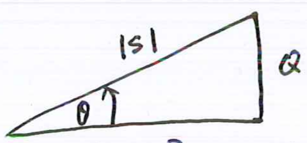
real power / average power — Watt

reactive power — VAR.

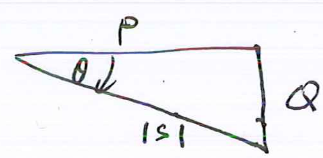
P: the measure of how much eletric ~~energy~~ power is converted to non-electric ~~energy~~ power. (heat, motion, ...)

Q: the measure of how much power goes to charge up all the reactive elements in a cirut. (such the magnetic field in a motor)

$|S|$: the measure of how much Volt-amp is required to supply in order to supply the P (make the work done)



Q is positive
current lags voltage.
$\theta_i - \theta_v > 0$
lagging (indutive) load



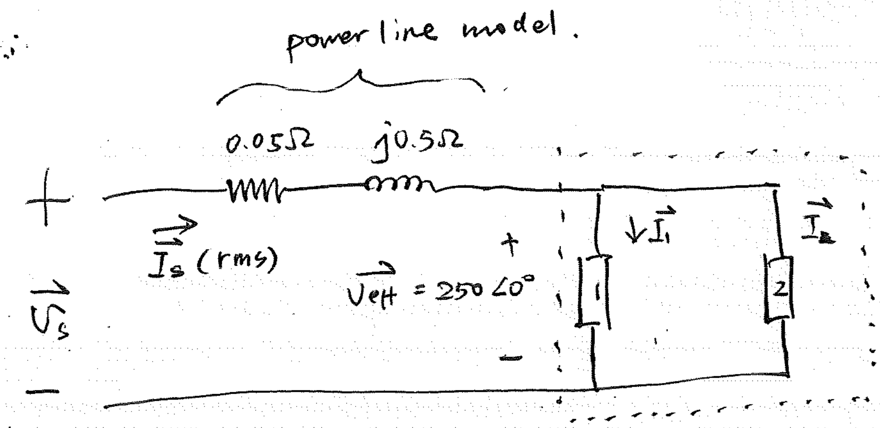
Q is negative
current leads voltage
$\theta_v - \theta_i > 0$.
Leading (capacitive) load

$$S = \overrightarrow{V_{eff}} \cdot \overrightarrow{I_{eff}}^* = \frac{1}{2} \vec{V} \cdot \vec{I}^*$$

$$P = |\overrightarrow{I_{eff}}|^2 R = \frac{1}{2} I_m^2 R \quad ; \quad Q = |\overrightarrow{I_{eff}}^*|^2 X = \frac{1}{2} I_m^2 X$$

a.

Example cont.:



We have already calculated that: (in the previous letture note)

The complex power consumed by the parallelly connected load, Load 1 + Load 2 is:

$$S = 20000 + j10000 \text{ VA}.$$

Now, we assume a simplified model of the power line, it has impedance of $0.05\Omega + j0.5\Omega$.

Find: ① The apparent power to support the loads.

② the magnitude of $\vec{I_s}$,

③ the average power lost in the transmission line.

$S = 20000 + j10000 \quad \Rightarrow |S| = \sqrt{(20K)^2 + (10K)^2} = \boxed{22.36 \text{ KVA}}$ ①

$\vec{I_s} = ?\qquad S = \vec{V_{eff}} \cdot \vec{I_{eff}}^{*} = \vec{V_{eff}} \cdot \vec{I_s}^{*} = 20K + j10K$

$$\vec{I_s}^{*} = \frac{20K + j10K}{250} = 80 + j40 \text{ A}.$$

$\vec{I_s} = 80 - j40 \text{ A}, \quad |\vec{I_s}| = \sqrt{80^2 + 40^2} = \boxed{89.44 \text{A}}$ ②

③ $P_{line} = |\vec{I_s}|^2 \cdot R = 89.44^2 \times 0.05 = \boxed{400 W}$ ③.

6. Remember that, all voltages and currents in this example is "rms".

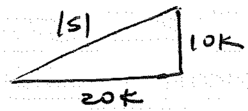
passive power factor correction. (PFC)

We have extra power loss due to the reactive powe.

We can use capacitor / inductor to cancel it!

we Know that the Load has:

$S = 20K + j10K$



$Q > 0$, inductive load.

$|S| = 22.36\ K\ V\cdot A$

if: we add sth to the parallel cirut, hopefully,

we will have: $S_{pfcd} = 20K + j10K \underline{\underline{-j10K}} = 20K.$

$\underline{S_{pfc} = -j10K}.$ $\quad Q_c = -10K = \dfrac{|\vec{V}_{eff}|^2}{X_c} = \dfrac{250^2}{X_C}$

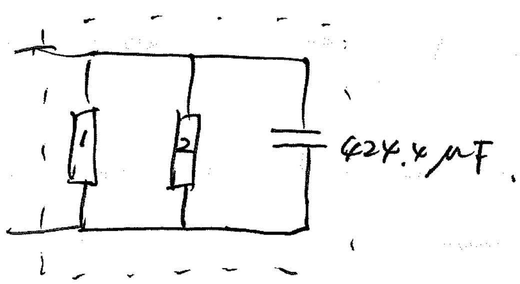
capacitor. $\quad \Rightarrow X_C = \dfrac{250^2}{-10K} = -6.25\Omega$

Remembe, $Z = R + jX$ for capacitor, $R = 0$. $X = \dfrac{-1}{wC}$

$-\dfrac{1}{wC} = -6.25, \quad w = 2\pi f = 2\pi \times 60$ rad.

$\Rightarrow C = \dfrac{1}{2\pi \times 60 \times 6.25} = 424.4\,\mu F.$

With the $424.4\,\mu F$ in parallel,



The final product:

$S = 20K$, real number,

$|S| = 20K\ V\cdot A, < 22.36\ KVA$

Then let's see how much power is lost in the transmission line

$\vec{I_s}^* = \dfrac{S}{\vec{V}_{eff}} = \dfrac{20K}{250} = 80 + j0\ A, \vec{I_s} = 80A.$ real!

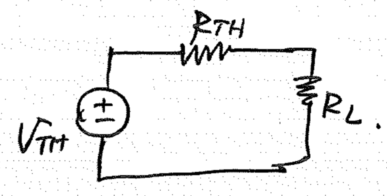
$P = |\vec{I_s}|^2 \cdot R = 80^2 \times 0.05 = 320W, < 400W.$ we save some power!

C.

Maximum Power Tranfer.

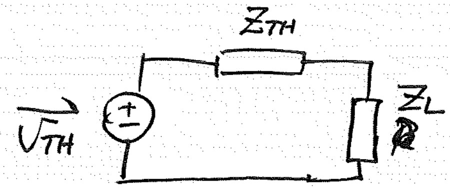
Systems that transmit information via electric signals depends on being able to deliver the maximum of power from the source to the load.

For DC circuit :



To get maximum power: $R_L = R_{TH}$.

For AC circut :



Let's see the conclution :

To get maximum power: $Z_{TH} = R + jx$.

$Z_L = R - jx$.

This is simplely because: $R_{TH} = R_L$. for the real part to exatrac the max power.

and for the imaginary part: just cancel it !

easy !

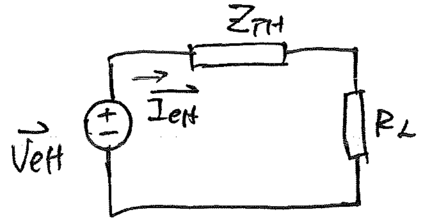
or: $Z_L = Z_{TH}^*$

So, if Z_{TH} is inductive, use capaitve load,

Z_{TH} is capactive, use indutive. load.

Then let's explain why.

d.



$$Z_{TH} = R_{TH} + j X_{TH}$$
$$Z_L = R_L + j X_L.$$

$$\vec{I}_{eff} = \frac{\vec{V}_{eff}}{R_{TH} + jX_{TH} + R_L + jX_L}$$

Let's look at the power delivered to the Z_L:

$$P = |\vec{I}_{eff}|^2 \cdot R_L = \frac{|\vec{V}_{eff}|^2 R_L}{(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2}$$

$\Rightarrow$ maximum?

$$\frac{\partial P}{\partial X_L} = \frac{-|\vec{V}_{eff}|^2 \cdot 2R_L(X_L + X_{TH})}{[(R_L+R_{TH})^2 + (X_L+X_{TH})^2]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{|\vec{V}_{eff}|^2 \cdot [(R_L+R_{TH})^2 + (X_L+X_{TH})^2 - 2R_L(R_L+R_{TH})]}{[(R_L+R_{TH})^2 + (X_L+X_{TH})^2]^2}$$

$$\frac{\partial P}{\partial X_L} = 0, \quad X_L = -X_{TH}. \quad \checkmark$$

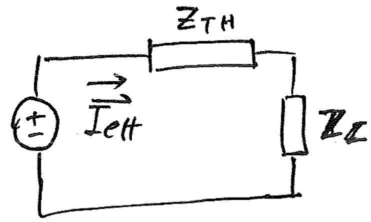
$$\frac{\partial P}{\partial R_L} = 0, \quad (R_L+R_{TH})^2 = 2R_L(R_L+R_{TH})$$

$$\Rightarrow R_L + R_{TH} = 2R_L \Rightarrow R_{TH} = R_L.$$

$$\Rightarrow \quad X_L = -X_{TH}, \quad R_L = R_{TH}.$$

$$Z_L = Z_{TH}^*$$

e.



$\vec{V}_{TH} \Rightarrow \vec{V}_{eff}$

Given $Z_L = Z_{TH}^*$. what is the maximum power transfered to the load?

$$\vec{I}_{eff} = \frac{\vec{V}_{eff}}{Z_{TH}+Z_L} = \frac{\vec{V}_{eff}}{2R_L}$$

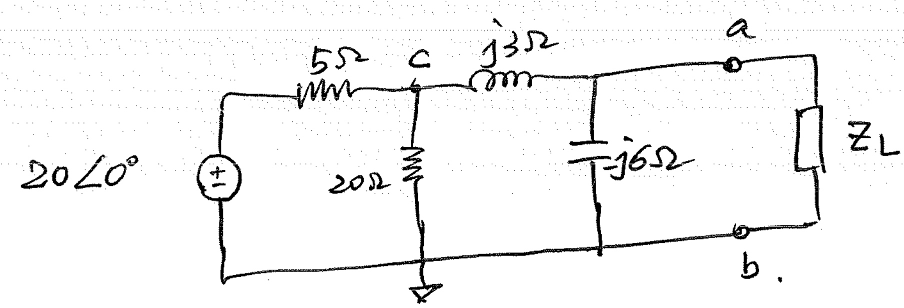
$$P = |\vec{I}_{eff}|^2 \cdot R_L = \frac{|\vec{V}_{eff}|^2}{4R_L^2} \times R_L = \frac{|\vec{V}_{eff}|^2}{4R_L}$$

If, V & I are in Amplitude form.

remember: $|\vec{V}_{eff}| = \frac{V_m}{\sqrt{2}}$, $|\vec{I}_{eff}| = \frac{I_m}{\sqrt{2}}$

$$P = \frac{|\vec{V}|^2}{8R_L} = \frac{V_m^2}{8R_L}$$

Example:



find Z_L for maximum power transfer.

① find V_{TH} & Z_{TH}.

$$\frac{\vec{V}_c - 20}{5} + \frac{\vec{V}_c}{20} + \frac{\vec{V}_c}{-j3} = 0. \Rightarrow \vec{V}_c = 5.76 - j7.68$$

$$\vec{V}_{ab} = \vec{V}_{TH} = \vec{V}_c \times \frac{-j6}{-j6+j3} = \vec{V}_c \times 2 = 11.52 - j15.36 = 19.2\angle -53.13° \ V.$$

(if just to find Z_L, we don't need to find $\vec{V}_{TH}$, though)

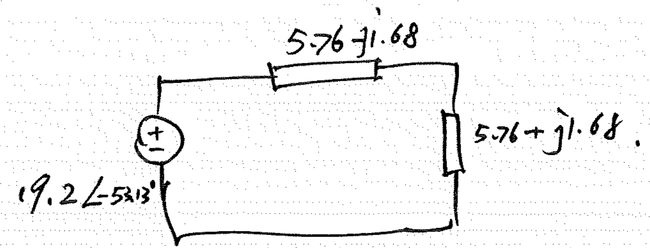
f

$$Z_{TH} \Rightarrow \quad [5 \parallel 20 + j3] \parallel (-j6)$$

$$\Rightarrow \frac{(4+j3)\times(-j6)}{4+j3-j6} = 5.76 - j1.68\ \Omega$$

$$Z_L = Z_{TH}^* = 5.76 + j1.68\ \Omega .$$

Then:



How much average power is delivered?

$$\vec{V}_{TH} = 19.2\angle -53.13^\circ, \quad |\vec{V}_{eff}| = \frac{19.2}{\sqrt{2}} = 13.58\ V.$$

$$P_{max} = \frac{|\vec{V}_{eff}|^2}{4R_L} = \frac{13.58^2}{4\times 5.76} = 8\ W.$$

g.