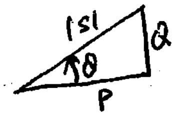
Complex Power Cont.

$S = P + jQ$

$\Uparrow$ average/real power

$\Nwarrow$ reative power.



lagging pf
$(\theta_v - \theta_i) > 0$

Q is negative?

leading pf.
$(\theta_v - \theta_i) < 0$

$$S = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2} \left[\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i) \right]$$

$$= \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i)$$

Using the rms or effective value, $V_{eff} = \frac{V_m}{\sqrt{2}}$, $I_{eff} = \frac{I_m}{\sqrt{2}}$

$$S = V_{eff} I_{eff} \angle(\theta_v - \theta_i)$$

We now have "S" in terms of phasers

$$S = V_{eff} I_{eff} e^{j\theta_v} \cdot e^{-j\theta_i}$$

$$= \underline{V_{eff} e^{j\theta_v}} \cdot I_{eff} e^{-j\theta_i}$$

$\Downarrow$ $\vec{V_{eff}}$

?? consider: $I_{eff} e^{-j\theta_i} = I_{eff} [\cos\theta_i - j \sin\theta_i]$

while: $\vec{I_{eff}} = I_{eff} e^{j\theta_i} = I_{eff} [\cos\theta_i + j \sin\theta_i]$

So, $I_{eff} e^{-j\theta_i} = \vec{I_{eff}}^*$. The complex conjugate of $\vec{I_{eff}}$.

$\Rightarrow S = \vec{V_{eff}} \cdot \vec{I_{eff}}^*$ (expressed in terms of rms, eff)

a.

Or, if using amplitude: $\vec{V_{eff}} = \frac{\vec{V}}{\sqrt{2}}$, $\vec{I_{eff}} = \frac{\vec{I}}{\sqrt{2}}$

$$S = \frac{1}{2}\vec{V}\cdot\vec{I}^*$$, still, $\vec{I}^*$ is the complex conjugate of $\vec{I}$.

Also, for complex power:

$$\boxed{S = |\vec{I_{eff}}^*|^2 \cdot Z} \qquad \boxed{S = \frac{|\vec{V_{eff}}|^2}{Z^*}}$$

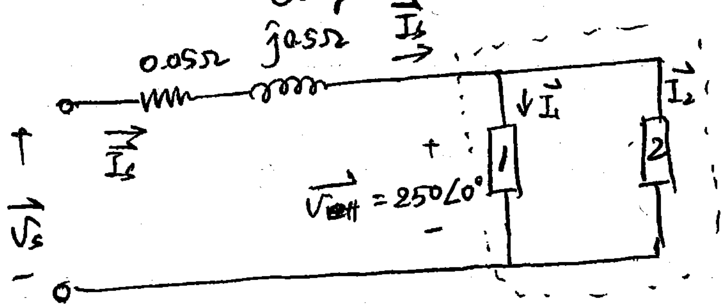
where $Z = R + jX$

So, it leads to: $P = |\vec{I_{eff}}|^2 \cdot R = \frac{1}{2}I_m^2 R$, $Q = |\vec{I_{eff}}^*|^2 \cdot X = \frac{1}{2}I_m^2 X$

While $P = \frac{|\vec{V_{eff}}|^2}{R} = \frac{V_m^2}{2R}$ is only true with a pure resistive component

$Q = \frac{|\vec{V_{eff}}|^2}{X} = \frac{V_m^2}{2X}$ is only true with a pure reactive component.

Example:



Load 1 absorbs an average power of 8KW at a leading power factor of 0.8; Load 2 absorbs 20K VA at a lagging power factor of 0.6.

Q. Determine the power factor of the 2 loads in parallel.

here, all currents & voltages are effective "rms" values.

From the schematic: $\vec{I_s} = \vec{I_1} + \vec{I_2}$.

The complex power of the parallel circuit:

$$S = \vec{V_{eff}} \cdot \vec{I_s}^* = 250\angle 0° \times (\vec{I_1} + \vec{I_2})^*$$

b:

$$= 250\angle 0^\circ \times \vec{I_1}^* + 250\angle 0^\circ \times \vec{I_2}^*$$

$$\underline{S = S_1 + S_2}$$

It's the sum of complex power of the two loads.

So, let's look at load 1.

We know $P_1 = 8KW$, a Leading power factor of 0.8

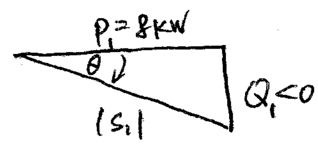
$\Downarrow$

$\theta_i - \theta_v > 0$, so $\theta = \theta_v - \theta_i < 0$

$\sin\theta < 0$.

$Q = \cdots \sin\theta < 0$.

draw the power triangler:



$\cos\theta = 0.8$, so: $|S_1| = \frac{8K}{0.8} = 10K$ VA.

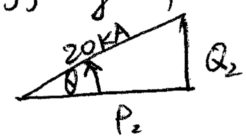
$Q_1 = -6K$ Var.

$$\boxed{S_1 = 8K - j6K}$$

Then let's look at load 2.

$|S_2| = 20K$ VA, lagging power factor of 0.6

apparent power.



$\theta_v - \theta_i > 0$, $\theta > 0$.

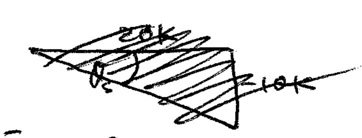
$Q_2 > 0$.

$\cos\theta = 0.6 \Rightarrow P_2 = 0.6 \times 20K = 12KW$.

$Q_2 = \sqrt{20K^2 - 12K^2} = 16K$ ~~Var~~ VAR

$$\boxed{S_2 = 12K + 16K}$$

$S = S_1 + S_2 = 20K + 10K$. power factor?



$\cos\theta = \frac{20K}{\sqrt{(20K)^2 + (10K)^2}} = 0.894$. (pf)

C.