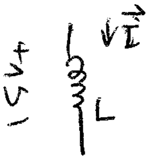
Power for Purely Inductive circuit.



For an inductor, the voltage leads the current by 90°. ⇒ $\theta_v = \theta_i + 90°$ ⇒ $\theta_v - \theta_i = 90°$.

$$P_{real} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = 0$$ → real power = 0 ?

$$\cancel{P}\; Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{V_m I_m}{2}$$ → reactive power ≠ 0.

So: $p(t) = P_{real} + P_{real}\cos(2wt) - Q\sin(2wt)$

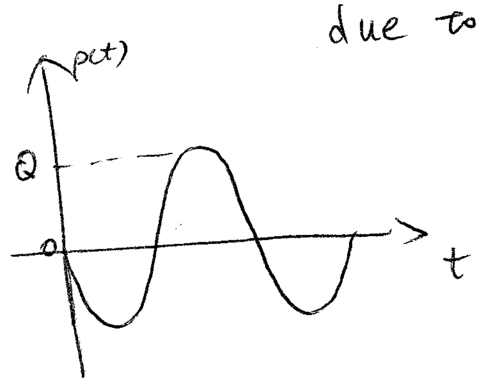
$$= -Q\sin(2wt) = -\frac{V_m I_m}{2}\,\cancel{\sin}\,\sin 2wt,$$

Observation: - In a pure inductive circuit, $\int_{t_0}^{t_0+T} p(t)\,dt = 0$, the energy consumption is "0".

- No electric energy is transformed to Non-electric energy.

- Power is exchanged, but not consumed between the inductive circuit and the source driving the circuit at a rate of 2w.

- In reality, since there are series resistance in ~~[illegible]~~, such as wires, so because of the AC current, there ~~may~~be energy consumed due to the resistance in the circuit.

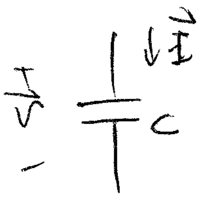


- when p(t) is positive, energy is being stored in the inductor (magnetic field)

- whe p(t) is negative, energy is being extracted from the indotor (magnetic field) and goes back to the sources.

a.

Power for the Purely Capacitive circuits.



In a capacitor, the current leads the voltage by 90°, $\theta_i = \theta_v + 90° \Rightarrow \theta_v - \theta_i = -90°$.

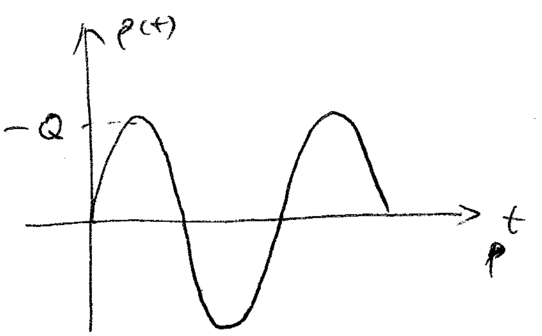
Similarly: $P_{real} = \frac{V_m I_m}{2} \cos(-90°) = 0$.

$$Q = \frac{V_m I_m}{2} \sin(-90°) = -\frac{V_m I_m}{2}$$

$$\Rightarrow p(t) = -Q \sin(2wt) = \frac{V_m I_m}{2} \sin 2wt.$$

$\int_{t_0}^{t_0+T} p(t)\, dt = 0$. no energy is consumed by capacitive circuit.

like inductors, Energy is exchanged between the source driving the capacitive circuit, & the eletric field of the capacitive circuit.



As for p(t), $P_{real} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$, $Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$

The angle "$\theta_v - \theta_i$" is referred to as the power factor angle.

$\cos(\theta_v - \theta_i) \leftarrow$ power factor (pf)

$\sin(\theta_v - \theta_i) \leftarrow$ reactive factor (rf).

usually, an eletronic productor will show their power factor, but knowing the power factor does not tell you the value of

b. the power factor angle, because,

$\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$

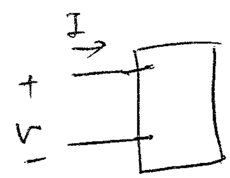
To completely describe this angle, engineers use the following describing terms:

Lagging power factor : current lags the voltage.
$\Rightarrow$ inductive load.

Leading power factor : current leads the voltage
$\Rightarrow$ Capacitive load.

The power factor or reactive factor are convenient ways to describe eletrical loads.

Example:



Assume: $v(t) = 100 \cos(\omega t + 15^\circ)$ V.
$i(t) = 4 \sin(\omega t - 15^\circ)$ A.

Find the average power & the reactive power.

First, the current is expressed in "sin", let's convert it to "cos" : $\sin\theta = \cos(\theta - 90^\circ)$

$$i(t) = 4\sin(\omega t - 15^\circ) = 4\cos(\omega t - 105^\circ)$$

Now, find P_{real} and Q.

$$P_{real} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{100 \times 4}{2} \times \cos[15^\circ - (-105^\circ)]$$

$$= 200 \times \cos 120^\circ = -100 \text{ W}.$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = 200 \times \sin 120^\circ = 173 \text{ Var}.$$

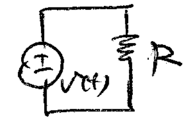
From passive sign convention, $P_{real} = \underset{\substack{\Uparrow \\ \text{negative.}}}{-100 \text{ W}}$.

The device is developing $\underset{\text{average}}{\wedge}$ power.

c.

rms power.

We have introduced this previously:

for $v(t) = V_m \cos(\omega t + \theta_v)$, if a load "R" is attached: 

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2\cos^2(\omega t+\theta_v)}{R}\,dt$$

$$= \frac{1}{R} \times \underbrace{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt}_{V_{rms}^2}$$

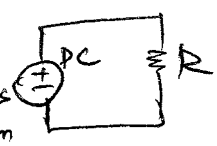
$$= \frac{V_{rms}^2}{R}$$

or: $p = I_{rms}^2 R$.

The power generated by $v(t) = V_m\cos(\omega t+\theta)$ delivered to R, is the same as a DC source with a value V_{rms}.

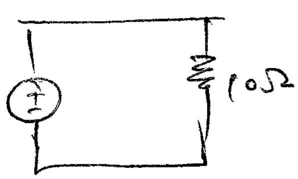
$v_s(t) = V_m\cos(\omega t+\theta)$ 

$\underline{\text{in terms of power}}$

$V = V_{rms}$ 

$= \frac{V_m}{\sqrt{2}}$

Example:

$\sqrt{2}\cos 10t$ 

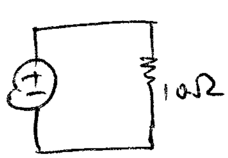
the $v(t) = \sqrt{2}\cos 10t$

$V_{rms} = \frac{\sqrt{2}}{\sqrt{2}} = 1V$.

$$P = \frac{V_{rms}^2}{R} = \frac{1^2}{10} = 0.1\,W.$$

Energy delivered to the load in 1S: $\int_0^1 0.1\,dt = 100mJ$

Same as

$1V$ 

, $P = \frac{V^2}{R} = 0.1W$.

energy in 1S: $\int_0^1 0.1\,dt = 100mJ$.

d.

The book sometimes refers to the Rms value as the <u>effective value</u>.

$$P_{real} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

can be rewritten as: $P_{real} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$

$$= V_{eff} \times I_{eff} \cos(\theta_v - \theta_i)$$

$$Q = V_{eff} I_{eff} \sin(\theta_v - \theta_i)$$

Complex Power.

The complex sum of average power and reactive power.

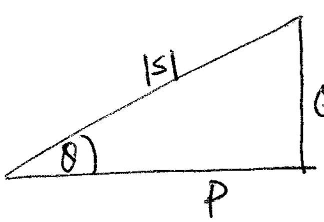
$$\boxed{S = P + jQ}$$

(here P_{real} we write it as P)

$P \Rightarrow$ Watt, $Q \Rightarrow$ Var

unit?

VA !! Volt - amp, (common for P & Q)



(Preal)

power triangle

$$|S| = \sqrt{P^2 + Q^2}$$

$$\theta = \theta_v - \theta_i, \quad \tan\theta = \frac{Q}{P}$$

Remember: $Q = \frac{I_m V_m}{2} \sin(\theta_v - \theta_i)$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$\frac{Q}{P} = \tan(\theta_v - \theta_i)$$

the term $|S|$ is referred to as <u>apparent power</u>.

e.

Usually, the apparent power is more important than the average power.

- . The ~~measur~~ average or real power is a measure of how much eletric energy to non-electric energy.

- . The reactive power is a measure of how much energy goes into charging up all of the reactive elements in a circuit.

- . The apparent power is a measure of how many Volt-amp is required from the supply in order to supply the average power

This is important because if :

desing A requires 100 VA to generate an average power of 10W.

design B requires 20VA to generate the same average power of 10W.

clealy, design B is a better design. (efficient)

the power triangle is a convenient tool to visualize how the average power, reative power, & the apparent power all relate.

For maximum efficiency we want the reactive power ~~H~~ to be as small as possible, (power factor to be as close ~~as~~ to 1 as possible)

f. If $\theta_v = \theta_i$, $\cos(\theta_v - \theta_i) = 1$, $pf = 1$, apparent power = average power, reactive power = 0.