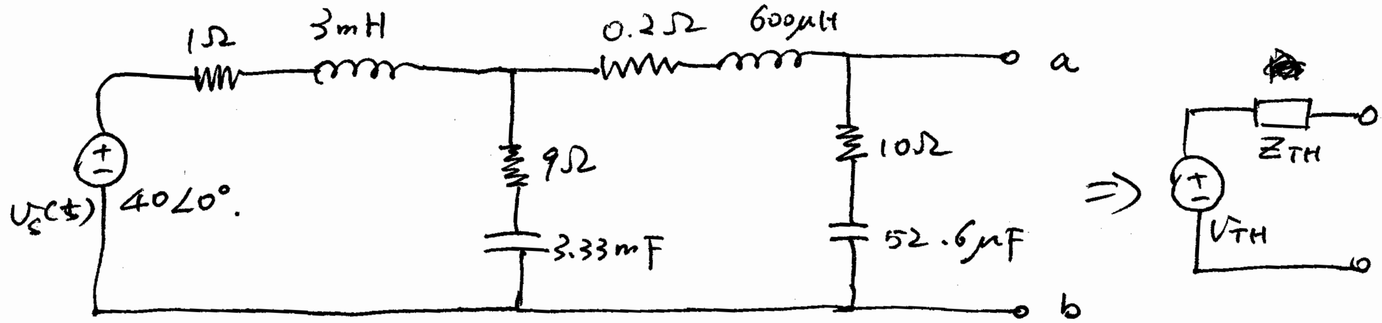
Source Transformation, Thevenim/Norton Equivalent circuit.

These are also valid when analysis AC circuit with Phasor.

In Frequency domain, to find Thevenin Equivalent circuit, the way of finding V_{TH} & R_{TH} is the same.
(Z_{TH} for Impedance)

Example:



The $V_s(t) = 40 \cos(1000t + 0°)$ V.
$\omega = 1000$ rad/s.

We can use any method we have learned: node-voltage method, Mesh-current method, ...)

Before continuing, let's convert the R, L, C into Z:

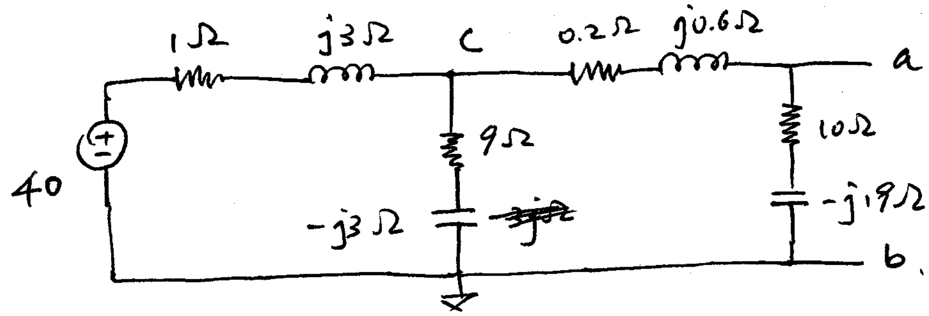
$Z_L = j\omega L$

$Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$

$3mH \Rightarrow j\omega \times L = j \times 1000 \times 3m = j3\Omega$ ... eventually:

The previous circuit can be redrawn as:

a.



Method 1 : Node - Volt - Meth.

We would like to find $\vec{V_a}$.

$$\frac{\vec{V_c} - 40}{1 + j3} + \frac{\vec{V_c}}{9 - j3} + \frac{\vec{V_c}}{10.2 - j18.4} = 0$$

Solve $\vec{V_c}$? (Tool: search "Simultaneous Linear Equation Complex"

click the 1st one :

keisan.casio.com/exec/system/1329101813.

it can solve up-to-4 unknowns.

Key in :

$$\boxed{\underbrace{\left(\frac{1}{1+3i} + \frac{1}{9-3i} + \frac{1}{10.2-18.4i}\right)}_{a_{11}}} \vec{V_c} = \boxed{\underbrace{\frac{40}{1+3i}}_{b_1}}$$

if you have multiple unknowns and equations, just key in,

$$\vec{V_c} = 35.784 - j17.688, = 39.9 \angle -26.3^\circ$$

$$\vec{V_a} = \frac{10 + -j19}{10.2 - j18.4} \cdot 39.9 \angle -26.3^\circ = 40.7 \angle -27.54^\circ = \vec{V_{TH}}.$$

Voltage divider eq.

b.

Search: "Simultaneous complex linear equation calculator".

Can have both $\vec{V_c}$ & $\vec{V_a}$:

$$\begin{cases} \dfrac{\vec{V_c}-40}{1+j3}+\dfrac{\vec{V_c}}{9-j3}+\dfrac{\vec{V_c}-\vec{V_a}}{0.2+j0.6}=0 \\ \dfrac{\vec{V_a}-\vec{V_c}}{0.2+j0.6}+\dfrac{\vec{V_a}}{10-j19}=0 \end{cases}$$

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Simultaneous equations (Complex) Calculator

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Calculates the solution of simultaneous linear equations with n variables. Variable are allowed input of complex numbers.

For input of complex number, please refer to "Description rule of expression"

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}=\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

(enter a data after click each cell in matrix)

matrix A $\{a_{ij}\}$

i\j	1	2	bi
1	1/(1+3i)+1/(9-3i)+1/(0.2+0.6i)	-1/(0.2+0.6i)	40/(1+3i)
2	-1/(0.2+0.6i)	1/(0.2+0.6i)+1/(10-19i)	0

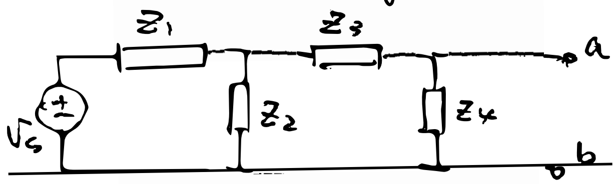
(Inc/Dec of the row/col) (Inc/Dec of the row)

Open Matrix Menu +

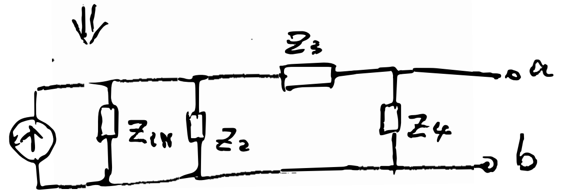
Execute · Clear · Store/Read · Print · 14dgt

x	solution	
x_1	35.784 -17.688i	$\vec{V_c}$
x_2	36.12 -18.84i	$\vec{V_a}$

Method 2. source transformation.



⇓



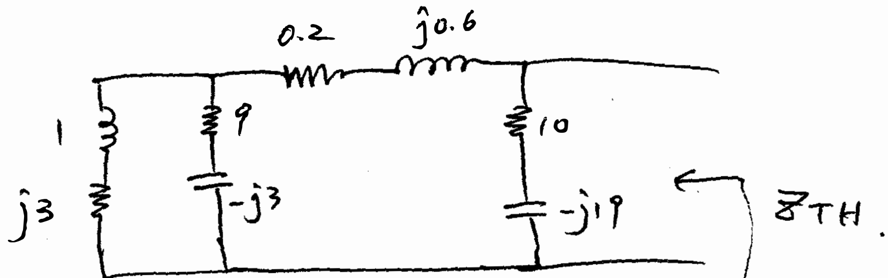
...

⇑

Remember: must keep Termial a, b., don't merge Z_3 & Z_4 to ~~to~~ find V_{ab}; merging Z_3 & ~~b~~ Z_4 ⇒ V_{ab} is gone (inside Z_3 & Z_4). You can find the current by merging Z_3 & Z_4, but Z_4 must be visible to find V_{ab}.

Then Find ~~RTH~~ Z_{TH}:

No dependent source? short, open.



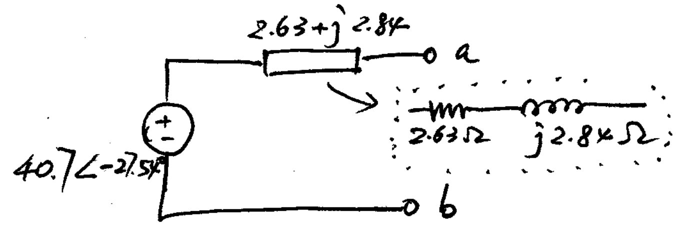
$$\Rightarrow Z_{TH} = \left\{ \underbrace{\left[(1+j3) // (9-j3) \right]} + (0.2 + j0.6) \right\} // 10 - j19$$

$$\frac{(1+j3)(9-j3)}{10} = \frac{18+j24}{10} = 1.8 + j2.4$$

...

$$= 2.63 + j2.84$$

So: Thevenin Eq:



Note that: other methods, such as super positioning, also works with AC source in freq. domain.

* Read Section 9.12 on phasor diagram *.

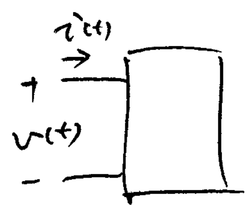
Steady-State Power Calculations.

Many electric energy is supplied in the form of sinusoidal voltages or current. We are usually interested in the average power delivered to the load. There are, also

reactive power
complex power
and apparent Power.

we will study these terms.

Instantanious power



At any time, $p(t) = v(t)\, i(t)$.

For a sinusoidal system:

$$v = V_m \cos(\omega t + \theta_v)$$

$$i = I_m \cos(\omega t + \theta_i)$$

Since we are considering steady-state, it does not matter when we start to look at the system, (choosing $t=0$).

d.

We can shift the time, so that the current is a cosine signal without phase shift.

$$v(t) = V_m \cos(wt + \theta_v - \theta_i)$$

$$i(t) = I_m \cos(wt).$$

So, $p(t) = v(t)i(t) = V_m I_m \cos(wt + \theta_v - \theta_i)\cos wt$

We can put this into a more informative form:

we know: $\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$

here assume: $\alpha = wt + \theta_v - \theta_i$, $\beta = wt$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2wt + \theta_v - \theta_i)$$

we know: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$p(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)} + \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2wt)} - \underbrace{\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2wt)}$$

$$p = P_{real} + P_{real}\cos(2wt) - Q\sin(2wt)$$

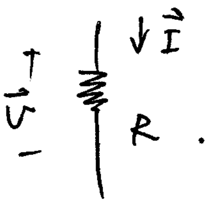
$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) \leftarrow$ reactive power. (Var)

$P_{real} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) \leftarrow$ real power. (Watt)

$\Downarrow$

often called real power because it describes electric energy used for doing real things (heat, light, motion ...)

Power for Purely Resistire cirurt.



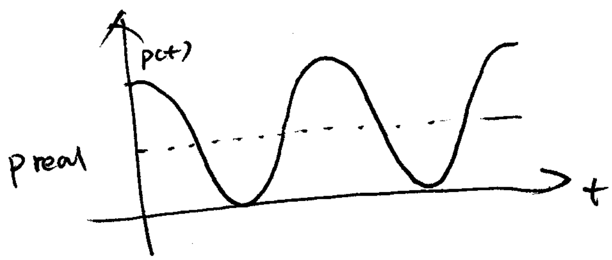
We know that the current and voltage has no phase different.

$$\theta_v = \theta_i.$$

$$P_{real} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m I_m}{2}$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = 0.$$

$$p(t) = P_{real} + P_{real} \cdot \cos 2\omega t.$$



instantanions power

f.