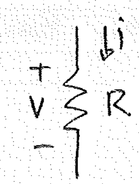
## The I-V Relationship for a Resistor:

From Ohm's Law we know that the voltage drop accross a resistor is <u>linearly</u> dependent upon the current.

This is still true even if the current is a function of time.

Assume: $i = I_m \cos(\omega t + \theta_i)$



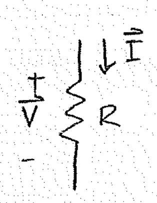

$$V = R\left[I_m \cos(\omega t + \theta_i)\right]$$

Now take the phasor transform of V.

$$\vec{V} = R I_m e^{j\theta_i} = R I_m \angle \theta_i$$

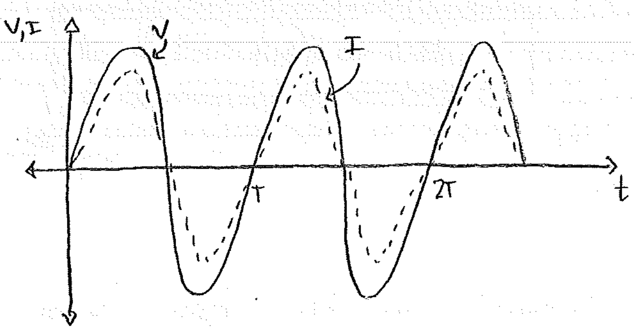
But we know that the phasor transform of $i$ is

$$\vec{I} = I_m \angle \theta_i$$




Therefore: $\boxed{\vec{V} = R\vec{I}}$

This equation tells us that there is no phase shift between the voltage and current at the terminals of the resistor. They are in-phase.




## The I-V Relationship for an Inductor.

In the time domain $v(t) = L\frac{di}{dt}$

If we assume a sinusoidal current:

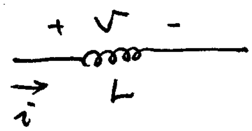
$$i = I_m \cos(\omega t + \theta)$$

(This page is from Prof. Nathan Neihart, another 201 instructor)

The IV Relation_ship for R, L, C.

For resisters, it ~~cont~~ only changes the amplitude term.
No change to phase.

$$\vec{V} = R \cdot \vec{I}, \qquad \vec{I} = \frac{\vec{V}}{R}.$$

Inductors: 

, we know: $v = L\frac{di}{dt}$

If we have $i(t) = I_m \cos(\omega t + \theta)$, $\vec{I} = I_m e^{j\theta}$

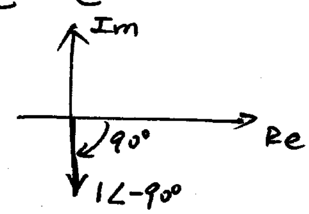
Then: $v(t) = L \cdot \frac{di}{dt} = -\omega L \sin(\omega t + \theta)$

We have a "sin" here, for easier analysis, convert it to "cos".

$$\Rightarrow v(t) = -\omega L I_m \cos(\omega t + \theta - 90^\circ)$$

Phasor transform: $\vec{V} = -\omega L I_m e^{j(\theta - 90^\circ)}$

$$= -\omega L I_m e^{j\theta} \cdot e^{-j90^\circ}$$

What is $e^{-j \cdot 90^\circ}$ ? $1\angle -90^\circ \rightarrow$ 


$= -j$.

or: $e^{-j90^\circ} = \cos 90^\circ - j \sin 90^\circ = -j$

$$\Rightarrow \boxed{\vec{V} = j\omega L \cdot \underbrace{I_m e^{j\theta}}_{\vec{I}} = j\omega L \cdot \vec{I}}$$

What is the significance of the "j" in "jωL" ?

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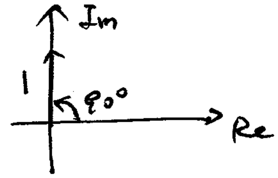
$$j = e^{j90^\circ} = 1\angle 90^\circ$$




Now, $\vec{v} = j\omega L \cdot I_m e^{j\theta} = j\omega L \cdot I_m \angle \theta^\circ$

rewrite:

$$\vec{v} = \omega L \cdot I_m e^{j(\theta + 90^\circ)} = \omega L \cdot I_m \angle (\theta^\circ + 90^\circ)$$

The amplitude difference is : "$\omega L$".

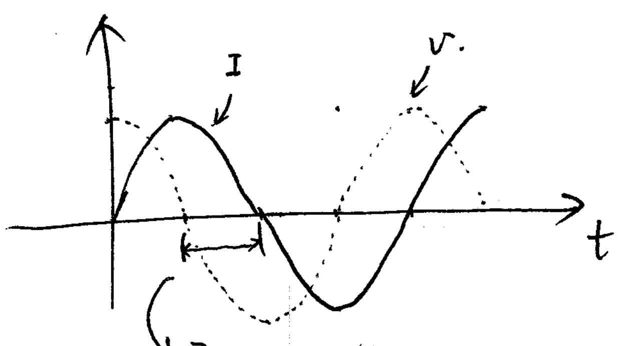
The phase difference is : $90^\circ$ or $\frac{\pi}{2}$.

Therefore, there is a $90^\circ$ phase shift between the voltage and current for an inductor!

$$i(t) = I_m \cos(\omega t + \theta)$$

$$v(t) = \omega L \cdot I_m \cdot \cos(\omega t + \theta \underline{\underline{+ 90^\circ}})$$

The voltage leads the current, or current lags the voltage, by $90^\circ$ or $\frac{\pi}{2}$.




note: $v$ reaches peaks earlier than $i$. "Lead".

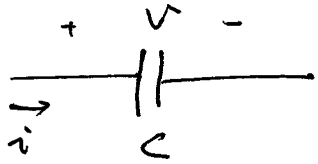
$\frac{\pi}{2}$ in radian

$90^\circ$ in degree

$\frac{T}{4}$ in time $\left( \omega = 2\pi f = \frac{2\pi}{T} . \quad \frac{\frac{T}{4}}{T} \times 2\pi = \frac{\pi}{2} \right)$ $\Rightarrow$ $\frac{\frac{\pi}{2}}{2\pi} \times T$ ( from rad to t )

$\frac{90^\circ}{360^\circ} \times T$ ( from degree to t )

Capacitor:




We know: $i(t) = C \cdot \frac{dv}{dt}$.

Now assume: $v(t) = V_m \cos(wt + \theta)$.

Using the same type of derivation that we were using for inductor, we can acheive:

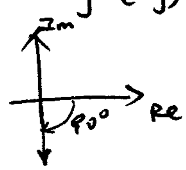
$$\vec{I} = \hat{j} w C \vec{V}.$$

$$\bar{V} = \frac{1}{jwC} \cdot \vec{I}$$

Now, we will look at the "$\frac{1}{j}$" term.

$$\frac{1}{j} = \frac{-j}{j \cdot (-j)} = -j.$$

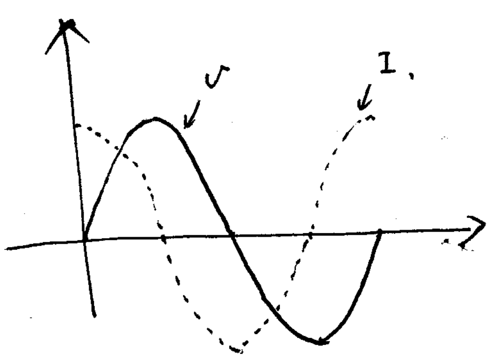
$-j \Rightarrow$




$e^{-j90°} = 1\angle -90°.$

$$\bar{V} = -j \cdot \frac{1}{wC} \cdot \vec{I} = \frac{1}{wC} \cdot \angle -90° . I_m \angle \theta° = \frac{1}{wC} I_m \angle \theta° - 90°$$

Again: we see a 90° phase shift. between the voltage and current of a capacitor.




In a cap, the current leads the voltage, or the voltage lags the current, by 90° ($\frac{\pi}{2}$, $\frac{T}{4}$)

Now, we have:

$$R \Rightarrow \vec{V} = R \cdot \vec{I}$$
$$L \Rightarrow \vec{V} = j\omega L \cdot \vec{I}$$
$$C \Rightarrow \vec{V} = \frac{1}{j\omega C} \cdot \vec{I}$$

They all have a similar form: $\vec{V} = Z \cdot \vec{I}$

Let's define: $Z$ is the "impedance", the impedance of an element is: the ratio of the voltage phasor to the current phasor. In all cases, $Z$ can be expressed in complex number: $Z = R + jX$. unit is $\Omega$ (Ohm's)

 $Z = R$ $\longrightarrow$ "$R$" $= R$, "$X$" $= 0$

 $Z = j\omega L$ $\longrightarrow$ "$R$" $= 0$, "$X$" $= \omega L$

 $Z = -\frac{j}{\omega C} = \frac{1}{j\omega C}$ $\longrightarrow$ "$R$" $= 0$, "$X$" $= -\frac{1}{\omega C}$

in: $Z = R + jX$

we call: $R \Downarrow$ resistance, $X \Rightarrow$ reactance.

Note that, passive sign conversion still applies, KVL, KCL still works.

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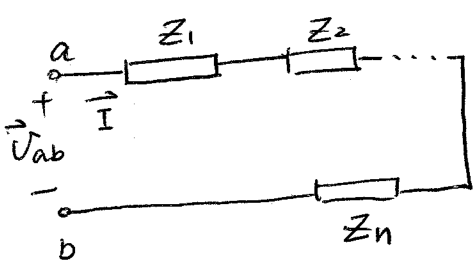
KVL: $\underbrace{v_1+v_2+\cdots v_n=0}_{\text{original}}$, $\underbrace{\vec{V}_1+\vec{V}_2+\cdots+\vec{V}_n=0}_{\text{phasor.}}$

KCL: $\underbrace{i_1+i_2+\cdots+i_n=0}_{\text{original}}$, $\underbrace{\vec{I}_1+\vec{I}_2+\cdots+\vec{I}_n=0}_{\text{phasor}}$

Important: there is an assumption here, the frequency must be the same!

## Series and Parallel Combination of Impedances.




The rules for combining series or parallel impedances is the same as those developed for resistors.

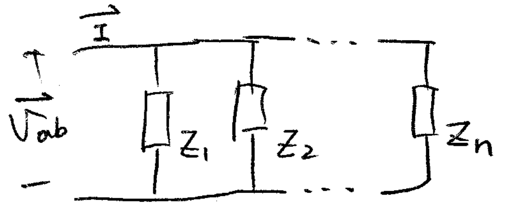
The only difference: $Z$ is a complex number

In series: current (either original form or phasor) must be the same.

$$\vec{V_{ab}} = Z_1\vec{I}_1 + Z_2\vec{I} + \cdots + Z_n\vec{I}$$
$$= \vec{I}_1 \cdot (Z_1+\cdots+Z_n)$$

$$\boxed{Z_{ab} = \sum_{i=1}^{n} Z_i.}$$ similar to resistor.

In Parallel:



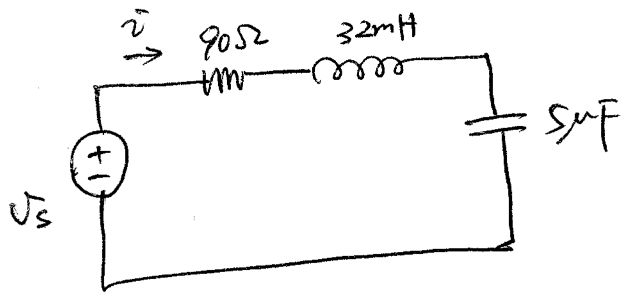

$$\frac{1}{Z_{ab}} = \sum_{i=1}^{n} \frac{1}{Z_i},$$ similar to resistor.

Like we define conductance: $G = \frac{1}{R}$.

We can define admittance: $Y = \frac{1}{Z} = G + jB$

$G$ ⇐ Conductance, $B$ ⇐ susceptance

Examples:




$V_s(t) = 750\cos(5000t + 30°)\,V$.

Find steady-state $i(t)$

First of all: $\omega = 5000$ rad/s $\quad V_m = 750\,V. \quad \phi = 30°$

$Z_R = 90, \quad Z_L = j\omega L = j \times 5000 \times 0.032 = j160\ \Omega$

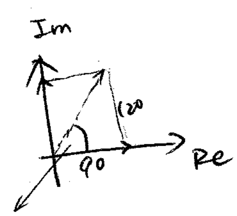
$Z_C = \frac{1}{j\omega C} = \frac{1}{j5000 \times 5 \times 10^{-6}} = -j40\ \Omega$

in series: $Z = 90 + j160 - j40 = 90 + j120$.

The phasor of $v(s) \Rightarrow 750\angle 30°$.

The phasor of $Z \Rightarrow 90 + j120 = 150\angle 53.13°$

$150 \Leftarrow \sqrt{90^2 + 120^2}$, $\quad 53.13° \Leftarrow \tan^{-1}\frac{120}{90}$



$$\vec{I} = \frac{\vec{V}}{Z} = \frac{750\angle 30^\circ}{150\angle 53.13^\circ} = \frac{750}{150}\angle 30^\circ - 53.13^\circ$$

$$= 5\angle -23.13^\circ$$

Multiplication

$$(A_1\angle\phi_1^\circ) \times (A_2\angle\phi_2^\circ) = (A_1 \times A_2)\angle(\phi_1^\circ + \phi_2^\circ)$$

Division

$$\frac{A_1\angle\phi_1^\circ}{A_2\angle\phi_2^\circ} = \frac{A_1}{A_2}\angle(\phi_1^\circ - \phi_2^\circ)$$

Addition:

$$A_1\angle\phi_1^\circ + A_2\angle\phi_2^\circ = (A_1\cos\phi_1^\circ + A_2\cos\phi_2^\circ) + j(A_1\sin\phi_1^\circ + A_2\sin\phi_2^\circ)$$

$$A_1\angle\phi_1^\circ \Downarrow A_1\cos\phi_1^\circ + jA_1\sin\phi_1^\circ$$

$$A_2\angle\phi_2^\circ \Downarrow A_2\cos\phi_2^\circ + jA_2\sin\phi_2^\circ$$

Then,

$$\sqrt{(A_1\cos\phi_1^\circ + A_2\cos\phi_2^\circ)^2 + (A_1\sin\phi_1^\circ + A_2\sin\phi_2^\circ)^2}\ \angle \tan^{-1}\frac{A_1\sin\phi_1^\circ + A_2\sin\phi_2^\circ}{A_1\cos\phi_1^\circ + A_2\cos\phi_2^\circ}$$